

# Phase reconstruction from intensity or amplitude measurements of the electromagnetic field

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# What is the phase problem?

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The phase problem is the name given to the problem of loss of information (the phase) from a physical measurement. The name itself comes from the field of x-ray crystallography, where the phase problem has to be solved for the determination of a structure from diffraction data. The phase problem is also met in the fields of imaging and signal processing. Various approaches have been developed over the years to solve it.

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Let  $S'(R)$  be the space of tempered distribution on  $R$  and set

$$A = \{g \in S'(R) : h \in L^1_{loc}(R)\}$$

Here  $h$  denotes the Fourier Transform, using the convention

$$h(k) = \int_{-\infty}^{+\infty} g(t)e^{ikt} dt$$

for  $g \in L^1(R)$ .

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Recall that for any  $g \in S'(R)$ , the Fourier transform is defined as another member of  $S'(R)$ . A first statement of the problem in which we are interested is as follows.

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## Phase determination problem.

Find  $g \in A$  given  $|h(k)|$  for  $k \in R$

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By the Fourier inversion theorem,  $g$  is uniquely determined by the complex valued function  $h(k)$  for  $k \in R$ , thus the essence of the matter is to determine the phase  $\phi(k) = \arg h(k)$  from the amplitude  $r(k) = |h(k)|$ .

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Problems of phase reconstruction arise in a number of interesting application areas. Now stated in this form, the problem is quite hopeless. If, for example, we fix any non-negative  $r \in L^2(\mathbb{R})$ , and

$$G(t) = \frac{1}{2\pi} \int_{-\infty}^{+\infty} r(k) e^{-i(\theta(k)+kt)} dk$$

then, for any measurable real valued function  $\theta$ , we have  $G \in L^2(\mathbb{R})$  and  $|H(k)| = r(k)$  on  $\mathbb{R}$ .



# Constrained phase determination problem

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## Constrained phase determination problem.

Find  $g \in B$  given  $|h(k)|$  for  $k \in R$

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If the constraint set  $B$  is sufficiently small, then one may hope that the phase of  $h$  is uniquely determined by its amplitude, or at least that the phase is constrained enough by the amplitude to considerably reduce the degree of non-uniqueness. Each different choice of admissible class  $B$  leads to a different problem with its own special features and difficulties. The admissibility criteria in the definition of  $B$  might be in the form of explicit conditions to be satisfied by the solution  $g$ , but could also incorporate other kinds of constraint, such as extra conditions related to the behaviour of  $h$ , which restrict the admissible solutions in an implicit way.

# The phase retrieval problem

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The phase retrieval problem is of wide interest because there are a variety of physically important problems in which essential quantities may be regarded as complex valued functions are thus characterized by amplitude and phase, where the phase is difficult or impossible to measure, while the amplitude is easily accessible to measurement or subject to some simple kind of inference. If knowledge of the phase is required for some reason, then we are faced with the constrained phase determination problem described in the first section, where the class  $B$  is defined in such a way as to incorporate constraints which are specific to each particular case.

# Applications in optics

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Such problems arise frequently in optics, where the complex valued function represents a propagating electromagnetic wave which is generally a function of position. When such a wave is scattered off an object, as in an electron microscope, then information about the shape of the object is propagated by the wave and is encoded in both the amplitude and the phase. Only the wave amplitude may be directly measured, and so one seeks to recover the lost phase information in order to characterize the object as thoroughly as possible. The underlying function  $g(t)$  (here  $t$  may be a two- or three-dimensional variable) can often be assumed to be a non-negative function of compact support.

# Applications in lens design

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A related problem in lens design has recently been studied. One is given a real non-negative function  $f(x)$ ,  $x \in \Omega \subset \mathbb{R}^2$  representing the transparency of a lens, and a desired far-field intensity pattern  $\Phi(\xi)$  and we seek to find a function  $\phi(x)$ ,  $x \in \Omega$ , representing the thickness of the lens so that the Fourier transform of  $f e^{i\phi}$  has amplitude  $\Phi$ . The constraint set  $B$  will then consist of functions  $g$  of compact support for which  $|g|$  is prescribed.

# X-ray crystallography

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One other important example arises in x-ray crystallography. Here one seeks to recover the electron density function  $\rho$  characterizing a certain crystal structure, which is taken to be a periodic function on  $R^3$ . In an x-ray diffraction experiment one measures the so-called structure factors, which are essentially the magnitude of the Fourier coefficients of  $\rho$  or, equivalently, the amplitude of the Fourier transform of  $\rho$  understood as a sum of delta functions supported on an appropriate lattice in  $R^3$ . The class  $B$  will consist of positive, periodic functions.

# X-ray crystallography

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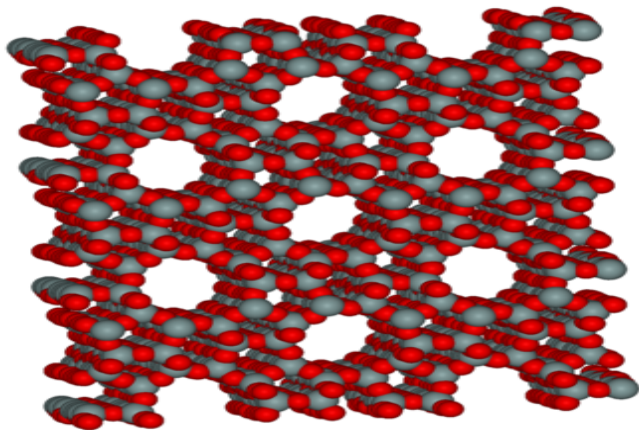
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# Setting of the problem

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Given  $Y \subset \mathbb{R}^3$ , a set of points;  $n$  and  $m$  are given integers, such that  $m \in \mathbb{N}$ ,  $n \in \mathbb{N}$ ,  $n \leq m$

$\phi_k$  is a family of functions:  $\phi_k : Y \rightarrow \mathbb{C} \quad k = 1, 2, \dots, n$

$Y_m$  is a set of  $m$  points:  $Y_m = \{x_p \in Y \quad p = 1, 2, \dots, m\}$



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$$e(u, x) = \sum_{k=1}^n u_k \phi_k(x)$$

$$h(u, x) = |e(u, x)|^2 = e(u, x) \overline{e(u, x)}$$

$$h(u, x) = \left( \sum_{k=1}^n u_k \phi_k(x) \right) \overline{\left( \sum_{k=1}^n u_k \phi_k(x) \right)}$$

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The problem is: given  $m$  positive real numbers  $\gamma_p$  such that  $\gamma_p \in R^+$ . Find  $n$  complex numbers  $u_k$  so that

$$h(u, x_p) = \gamma_p \quad p = 1, 2, \dots, m$$

In order to solve the problem we should minimize the following functional:

$$J(u) = \sum_{p=1}^m (h(u, x_p) - \gamma_p)^2$$

# What we should do?

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For that we should perform the following steps:

- 1) Get the expressions  $\frac{\partial J}{\partial \alpha_l}$
- 2) Get the descent direction  $d$  associated with Fletcher-Reeves modification
- 3) Use Newton method to find the minimum in the descent direction

# The 1st step

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First of all, we should write the expression of the derivative of the cost function with respect to the real parts of the vector  $u$

$$\frac{\partial J}{\partial \alpha_R} = 2 \sum_{p=1}^m (h(u, x_p) - \gamma_p) \frac{\partial h}{\partial \alpha_R}$$

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where the derivatives of  $h$  with respect to the real parts of the vector  $u$  are as the following

$$\frac{\partial h}{\partial \alpha_R} = \frac{\partial}{\partial \alpha_R} (z\bar{z}) = \frac{\partial z}{\partial \alpha_R} \bar{z} + z \frac{\partial \bar{z}}{\partial \alpha_R}$$

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where the derivatives of the auxiliary function  $z$  with respect to the real parts of the vector  $u$  are as the following

$$\frac{\partial z}{\partial \alpha_R} = \frac{\partial}{\partial \alpha_R} \left[ \sum_{k=1}^n (\alpha_{2k-1} + i\alpha_{2k}) \phi_k(x) \right] = \phi_{\frac{R}{2}+1}(x)$$
$$\frac{\partial \bar{z}}{\partial \alpha_R} = \overline{\phi_{\frac{R}{2}+1}(x)}$$

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The expression of the derivative of the cost function with respect to the imaginary parts of the vector  $u$

$$\frac{\partial J}{\partial \alpha_R} = 2 \sum_{p=1}^m (h(u, x_p) - \gamma_p) \frac{\partial h}{\partial \alpha_I}$$

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where the derivatives of  $h$  with respect to the imaginary parts of the vector  $u$  are as the following

$$\frac{\partial h}{\partial \alpha_I} = \frac{\partial}{\partial \alpha_I} (z\bar{z}) = \frac{\partial z}{\partial \alpha_I} \bar{z} + z \frac{\partial \bar{z}}{\partial \alpha_I}$$



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where the derivatives of the auxiliary function  $z$  with respect to the imaginary parts of the vector  $u$  are as the following

$$\frac{\partial z}{\partial \alpha_I} = \frac{\partial}{\partial \alpha_I} \left[ \sum_{k=1}^n (\alpha_{2k-1} + i\alpha_{2k}) \phi_k(x) \right] = i \phi_{\frac{I+1}{2}}(x)$$

$$\frac{\partial \bar{z}}{\partial \alpha_I} = -i \overline{\phi_{\frac{I+1}{2}}(x)}$$

# Conjugate gradient method

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The class of conjugate direction methods can be viewed as being intermediate between the methods of steepest descent and Newton's method. The conjugate direction methods have the following properties:

1. Solve quadratics of  $n$  variables in  $n$  steps
2. The usual implementation, the conjugate gradient algorithm, requires no Hessian matrix evaluations
3. No matrix inversion and no storage of an  $n \times n$  matrix required

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1. Select the initial point  $u^{(0)}$

$$2. g^{(0)} = \nabla J(u^{(0)})$$

If  $g^{(0)} = 0$ , stop, else set  $d^{(0)} = -g^{(0)}$

$$3. \alpha_k = -\frac{g^{(k)T} d^{(k)}}{d^{(k)T} Q d^{(k)}}$$

$$4. u^{(k+1)} = u^{(k)} + \alpha_k d^{(k)}$$

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$$5. \quad g^{(k+1)} = \nabla J(u^{(k+1)})$$

If  $g^{(k+1)} = 0$ , stop

$$6. \quad \beta_k = \frac{g^{(k+1)T} Q d^{(k)}}{d^{(k)T} Q d^{(k)}}$$

$$7. \quad d^{(k+1)} = -g^{(k+1)} + \beta_k d^{(k)}$$

8. Go to step 3

# The 2nd step

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## 1. The Hestenes-Stiefel formula

$$\beta_k = \frac{g^{(k+1)T} [g^{(k+1)} - g^{(k)}]}{d^{(k)T} [g^{(k+1)} - g^{(k)}]}$$

## 2. The Polak-Ribiere formula

$$\beta_k = \frac{g^{(k+1)T} [g^{(k+1)} - g^{(k)}]}{g^{(k)T} g^{(k)}}$$

## 3. The Fletcher Reeves formula

$$\beta_k = \frac{g^{(k+1)T} g^{(k+1)}}{g^{(k)T} g^{(k)}}$$

# The 3rd step

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In order to find the minimum in the descent direction we need to solve the following problem

$$\alpha_k = \arg \min_{\alpha \geq 0} J(u^{(k)} + \alpha d^{(k)})$$

# Newton method

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Observe that the  $k$ -th iteration of Newton's method can be written in two steps as

1. Solve  $F(u^{(k)})d^{(k)} = -g^{(k)}$  for  $d^{(k)}$
2. Set  $u^{(k+1)} = u^{(k)} + d^{(k)}$

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To avoid confusions let us use  $\xi$  instead of  $\alpha$ . So we need to solve the following problem:

$$\xi_k = \arg \min_{\xi \geq 0} J(u^{(k)} + \xi d^{(k)})$$

Assume that

$$v^{(k)} = u^{(k)} + \xi d^{(k)}$$

$$J(v^{(k)}) = 2 \sum_{p=1}^m \left( h(v^{(k)}, x_p) - \gamma_p \right)^2$$



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Newton's method requires computing the gradient of  $J$  with respect to  $\xi$  on each iteration

$$\frac{\partial J(v^{(k)})}{\partial \xi} = 2 \sum_{p=1}^m \left( h(v^{(k)}, x_p) - \gamma_p \right) \frac{\partial h(v^{(k)}, x_p)}{\partial \xi}$$

The derivative of the function  $h$  with respect to  $\xi$  is

$$\frac{\partial h(v^{(k)}, x_p)}{\partial \xi} = \frac{\partial \theta}{\partial \xi} \bar{\theta} + \frac{\partial \bar{\theta}}{\partial \xi} \theta$$

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The auxiliary function  $\theta$  is

$$\theta = \sum_{k=1}^n v^{(k)} \phi_k(x)$$

The first derivative of  $\theta$  with respect to  $\xi$  is

$$\frac{\partial \theta}{\partial \xi} = \frac{\partial}{\partial \xi} \sum_{k=1}^n v^{(k)} \phi_k(x) = \sum_{k=1}^n d_k \phi_k(x)$$

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Its complex conjugate is

$$\frac{\partial \bar{\theta}}{\partial \xi} = \frac{\partial}{\partial \xi} \sum_{k=1}^n \overline{v^{(k)}} \overline{\phi_k(x)} = \sum_{k=1}^n \overline{d_k} \overline{\phi_k(x)}$$

Newton's method also requires computing of the second derivative of  $J$  with respect to  $\xi$

$$\frac{\partial^2 J}{\partial \xi^2} = 2 \sum_{p=1}^m \left( \frac{\partial h(v^{(k)}, x_p)}{\partial \xi} \right)^2 + 2 \sum_{p=1}^m \left( h(v^{(k)}, x_p) - \gamma_p \right) \frac{\partial^2 h}{\partial \xi^2}$$

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The second derivative of the function  $h$  with respect to  $\xi$

$$\frac{\partial^2 h(v^{(k)}, x_p)}{\partial \xi^2} = 2 \frac{\partial \theta}{\partial \xi} \frac{\partial \bar{\theta}}{\partial \xi} = 2 \left| \frac{\partial \theta}{\partial \xi} \right|^2$$

The  $k$ -th iteration of the Newton's method is as follows

$$\xi_k = \xi_{k-1} - \frac{\partial J(\xi_{k-1})}{\partial \xi} \left( \frac{\partial^2 J(\xi_{k-1})}{\partial \xi^2} \right)^{-1}$$

# Microsoft Visual Studio

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Microsoft Visual Studio is an Integrated Development Environment (IDE) from Microsoft. It can be used to develop console and graphical user interface applications along with Windows Forms applications, web sites, web applications, and web services in both native code together with managed code for all platforms supported by Microsoft Windows, Windows Mobile, Windows CE, .NET Framework, .NET Compact Framework and Microsoft Silverlight.

# Microsoft Visual Studio

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Visual Studio includes a code editor supporting IntelliSense as well as code refactoring. The integrated debugger works both as a source-level debugger and a machine-level debugger. Other built-in tools include a forms designer for building GUI applications, web designer, class designer, and database schema designer. It allows plug-ins to be added that enhance the functionality at almost every level - including adding support for source control systems (like Subversion and Visual SourceSafe) to adding new toolsets like editors and visual designers for domain-specific languages or toolsets for other aspects of the software development lifecycle (like the Team Foundation Server client: Team Explorer).

# Library "complex"

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In the problem we need to handle with complex-valued functionals. Microsoft Visual Studio gives the possibility to work with them easily.

The standard library `<complex>` defines the container template class `complex` and its supporting templates.

# Library "complex"

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The most important functions of the library are

1. *abs()* (extracts the modulus of a complex number)
2. *conj()* (returns the complex conjugate of a complex number)
3. *exp()* (returns the exponential function of a complex number)
4. *imag()* (extracts the imaginary component of a complex number)
5. *real()* (extracts the real component of a complex number)



# Library "complex"

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The construction `complex < double >` describes an object that stores an ordered pair of objects both of type `double`, the first representing the real part of a complex number and the second representing the imaginary part.

The explicit specialization of the template class `complex` to a complex class of type `double` differs from the template class only in the constructors it defines. The conversion from `float` to `double` is allowed to be implicit, but the conversion from `long double` to `double` is required to be explicit. The use of explicit rules out the initiation with type conversion using assignment syntax

# Example

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```
#include <complex>
#include <iostream>

int main( )
{
    using namespace std;
    double pi = 3.14159265359;

    // The first constructor specifies real & imaginary ←
    // parts
    complex <double> c1 ( 4.0 , 5.0 );
    cout << "Specifying initial real & imaginary parts, \←
    n"
    << " as type double gives c1 = " << c1 << endl;
```

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```
// The second constructor initializes values of the ←  
    real &  
    // imaginary parts using those of complex number of ←  
    type float  
complex <float> c2float ( 4.0 , 5.0 );  
complex <double> c2double ( c2float );  
cout << "Implicit conversion from type float to type ←  
    double,"  
    << "\n gives c2double = " << c2double << endl;  
// The third constructor initializes values of the ←  
    real &  
    // imaginary parts using those of a complex number  
    // of type long double  
complex <long double> c3longdouble ( 4.0 , 5.0 );  
complex <double> c3double ( c3longdouble );
```

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```
cout << "Explicit conversion from type float to type ←  
double,"  
    << "\n gives c3longdouble = " << c3longdouble ←  
    << endl;  
  
// The modulus and argument of a complex number can ←  
be recovered  
double absc3 = abs ( c3longdouble );  
double argc3 = arg ( c3longdouble );  
cout << "The modulus of c3 is recovered from c3 ←  
using: abs ( c3 ) = "  
    << absc3 << endl;  
cout << "Argument of c3 is recovered from c3 using:\n ←  
n arg ( c3 ) = "  
    << argc3 << " radians , which is " << argc3 * ←  
    180 / pi  
    << " degrees." << endl;
```

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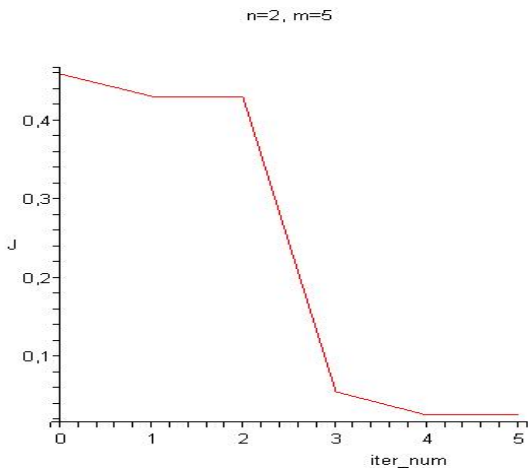
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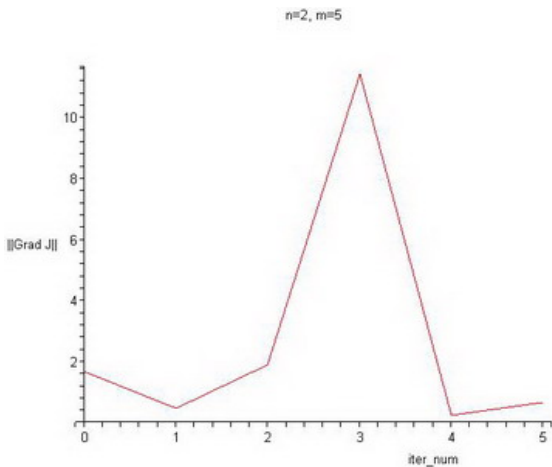
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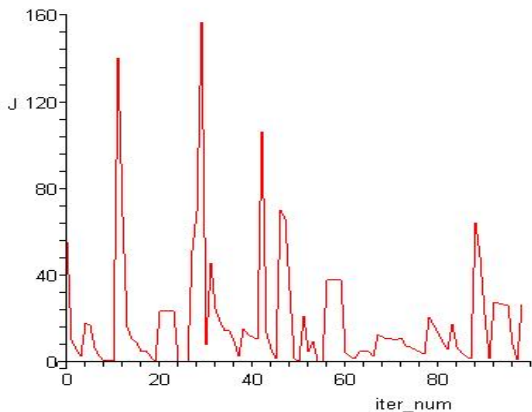
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$n=2, m=30$



# Results

Phase reconstruction from intensity or amplitude measurements of the electromagnetic field

Alexey A. Novikov

Outline

Introduction

Examples

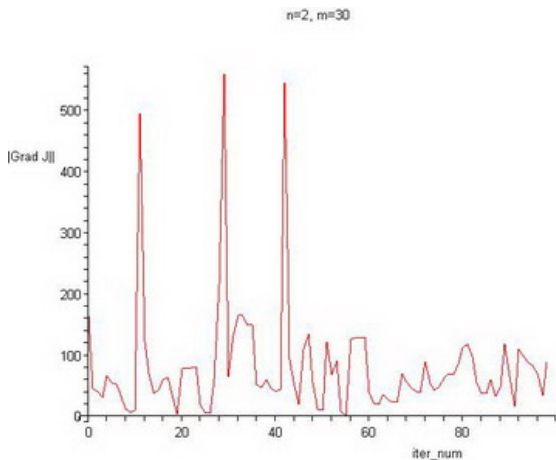
Mathematical model

Optimization Methods

Implementation And Technologies

Results

Conclusion





# How to improve results?

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1. Minimize another cost function
2. Find better method of finding the descent direction
3. Choose another method