# Numerical experiments on resistance optimal strategy

## Training in Industry - Mathmods - INRIA, Project TOSCA

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# Outline

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# Background

Three approaches can be used in analysing the financial market: the fundamental approach, the mathematical approach and the technical analysis. The last one involves analysing the charts and the transaction volumes, and it is widely used by traders.

In technical analysis, it is assumed that the past records of prices and transaction volumes can fully desribes the future trend, which contradicts the mathematical approach, because the latter one have the hypothesis that the prices form a Markov chain which means that future states depend only on the present state, and are independent of past states.

# Background

One classica pattern of charts in technical analysis is that the price moves between two barrier levels in a phase of consolidation. The upper one is called resistance, the lower one is called support(See[1]). When the price touches the support for three time, we can expect it to rise and the price will go up through the resistance. Though it does not have mathematical justification, this belief will have an influence on traders' behavior, and therefore on the price.

In[4], Bergery, Profeta & Tanré propose a mathematical model derived from the classic Black and Scholes model which has resistance. The trajectory makes a random number of downcrossings between two fixed levels before leaving.

# The work

The work that we have been doing involves implementing three strategies for classic Black Scholes, technical analysis and the newly derived model, respectively, simulating the price paths and comparing the efficiency of different strategies.

# Classic Black Scholes Model

We recall that together with other assumptions, the price of risky asset follows a geometric Brownian motion with constant drift and volatility, i.e. it is the solution of the stochastic differential equation:

$$\frac{dS_t}{S_t} = \mu_0 dt + \sigma dB_t \tag{1}$$

i.e.

$$S_t = S_0 \exp(\sigma B_t + (\mu_0 - \frac{\sigma^2}{2})t) = S_0 \exp(\sigma(B_t + \mu t)) = S_0 \exp(\sigma B_t^{\mu}) \tag{2}$$

where $\mu := \frac{1}{\sigma}(\mu_0 - \frac{\sigma^2}{2})$ and $B_t^{\mu} := B_t + \mu t$.

# The Newly Derived Model

In the previous model, the risky asset does not present support or resistance levels. And here, I will explain how the new model was constructed such that it has the property of a resistance level.

**Some Definitions**

- $B_t^\mu := B_t + \mu t, \;\; t \geq 0, \;\; \mu \neq 0$
- $\epsilon > 0$, $\alpha > 0$, two positive reals (fixed levels)
- $A_n$ : "$B^\mu$ has done at least n downcrossings from 0 to $-\alpha$ before hitting the level $\epsilon$"
- $M_t^n := \mathbb{P}(A_n|\mathcal{F}_t)$, where $\mathcal{F}_t$ is the natural filtration of $B_t^\mu$.
- For convenience, we let $\sigma_0 = \sigma_{-1} = 0$, and
  $\sigma_{2k+1} = \inf\{t \geq \sigma_{2k}; B_t^\mu = -\alpha\}$,
  $\sigma_{2k+2} = \inf\{t \geq \sigma_{2k+1}; B_t^\mu = 0\}$.

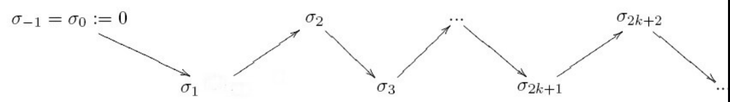



Figure: Downcrossings


Figure: a sample path of the brownian motion with drift

# What is $\epsilon$ and $\alpha$

Actually, if we let $S_0$ be the starting price of the risky asset and $S_0^-$, $S_0^+$ be two fixed levels with $0 < S_0^- < S_0 < S_0^+$
Since $S_t = S_0 \exp(\sigma B_t^\mu)$ then actually

$$\alpha = -\frac{1}{\sigma}\log(\frac{S_0^-}{S_0}) \quad (3)$$

$$\epsilon = \frac{1}{\sigma}\log(\frac{S_0^+}{S_0}) \quad (4)$$

So the downcrossings of process S from level $S_0$ to level $S_0^-$ are exactly the downcrossings of the process $B_t^\mu$ from level 0 to level $-\alpha$.

# The Newly Derived Model

- $t \geq \sigma_{2n-1}$, then $X$ is a standard Brownian motion with drift $\mu$ starting at -$\alpha$ at time $\sigma_{2n-1}$
- if $t \in [\sigma_{2i-1}, \sigma_{2i}]$, $i \in [0, n-1]$, then $X$ is a standard Brownian motion with drift $\mu$ starting at -$\alpha$ at time $\sigma_{2i-1}$
- if $t \in [\sigma_{2i}, \sigma_{2i+1}]$, $i \in [0, n-1]$, then $X$ is a solution of the equation

$$dX_t = d\widetilde{B_t} - \mu \coth(\mu(\epsilon - X_t))dt \tag{5}$$

We notice that

$$\coth x = \frac{e^{2x}+1}{e^{2x}-1} \tag{6}$$

So when $X_t$ goes near the $\epsilon$, this term tends to be very large, and it would cause the $X_t$ to go down sharply.
Then the price process is $Z = S_0 e^{\sigma X}$.

**Theorem**

*The law of the process $Z$ (resp. $X$) is the same as the law of $S$ (resp. $B^{\mu}$) conditioned by $A_n$.*

With this result , the expected wealth for the logarithmic utility function is $\mathbb{P}(W^{\pi}|A_n)$ and it can be explicitly calculated.

# Implementation

Now the problem we face is how to implement the path generation into the computer.
By the definition of the standard brownian motion

$$B^t - B^s \sim \mathcal{N}(0, t - s) \quad (7)$$

So what we did is to generate a gaussian number with mean 0 and variance 1, and then

$$B_\mu[i+1] = B_\mu[i] + \mu dt + \sqrt{dt} * gaussiannumber \quad (8)$$

Now we obtain the standard Brownian motion with drift

# Moving on

More challenging work is to generate the path during downcrossing. We look at the design of the trajectory:

$$dX_t = d\widetilde{B_t} - \mu \coth(\mu(\epsilon - X_t))dt \tag{9}$$

First, we tried discretizing it using a explicit method, i.e.

$$X_t[i+1] - X_t[i] = dB_t - \mu \coth(\mu(\epsilon - X_t[i]))dt \tag{10}$$

But this scheme turned out to be not stable when $X_t$ near $\epsilon$ Then, we adopted the implicit scheme, it worked very well!

$$X_t[i+1] - X_t[i] = dB_t - \mu \coth(\mu(\epsilon - X_t[i+1]))dt \tag{11}$$

# Simulation of the density function

After obtaining the data from our simulations, we need to check if the data are really what we want. Here, I need some knowledge from computational probability.

*Simulation of the density function*

When we want to simulate the density function, a very popular way is to use the kernel method:

$$f^h(t) = \frac{1}{NMC\sqrt{2\pi h}} \sum_{i=1}^{NMC} \exp\left(-\frac{(p^i - t)^2}{2h}\right) = v^{NMC} \otimes K^h \qquad (12)$$

Where $v^{NMC} = \frac{1}{NMC} \sum\limits_{i=1}^{NMC} \delta_{x^i}$ and $K^h$ is the Gaussian kernel with parameter h.

In this section, we are trying to find the dependency of the density function simulation on parameter h. In order to do this, we start our numerical test with very small value of h, and see how the graph of the density function would change corresponding to different values of h.

We did all the experiments under the fixed Monte Carlo Simulation Number=10000.

Red line- Density function simulation
Blue line- Real density function

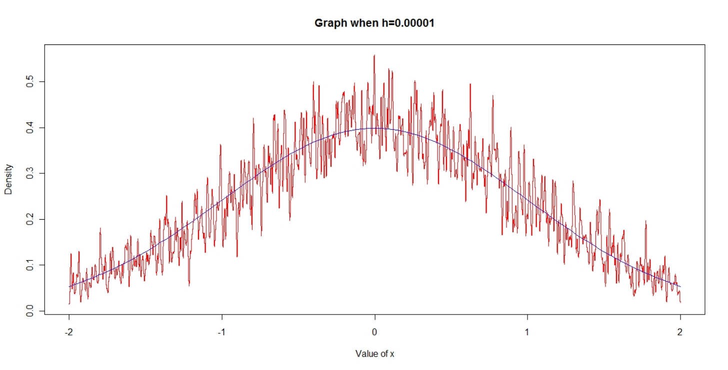



Figure: h=0.00001

Red line- Density function simulation
Blue line- Real density function

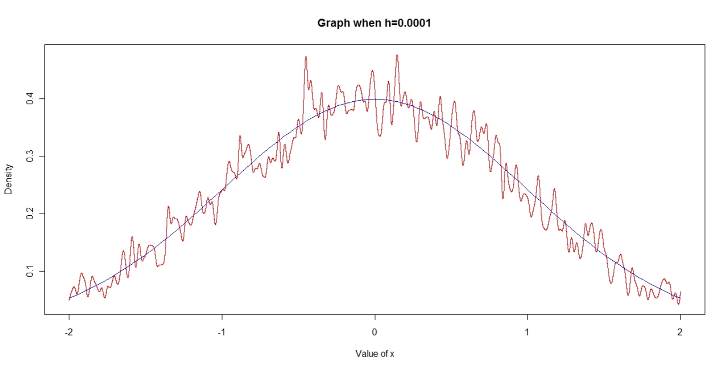



Figure: h=0.0001

Red line- Density function simulation
Blue line- Real density function

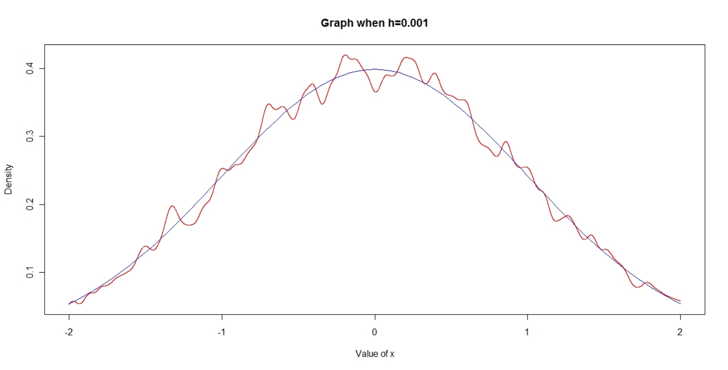



Figure: h=0.001

Red line- Density function simulation
Blue line- Real density function

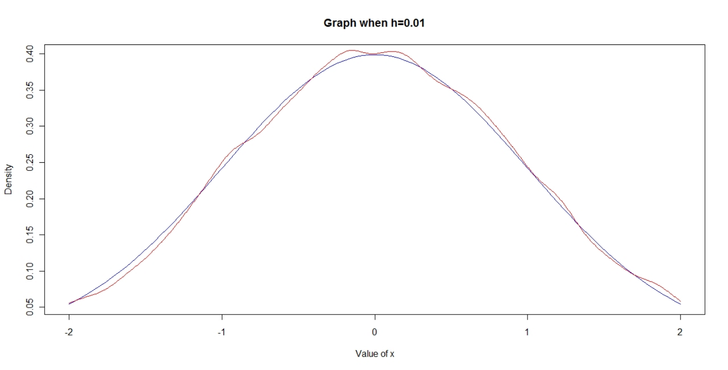



Figure: h=0.01

Red line- Density function simulation
Blue line- Real density function

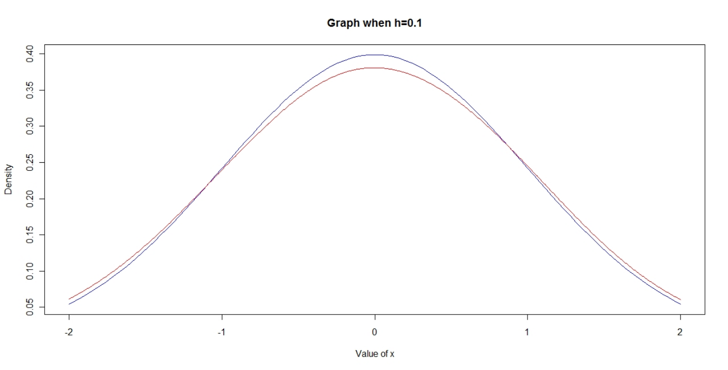



Figure: h=0.1

Red line- Density function simulation
Blue line- Real density function

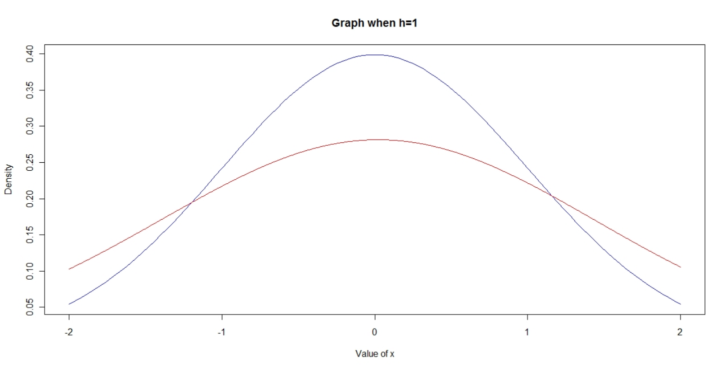



Figure: h=1

So as we can see from the graphs, the order of $h$ could affect the simulation strongly. Firstly, by the theorem that $f \otimes K^h \stackrel{h \to 0}{\longrightarrow} f$, where $f$ is the density function. so h should be kept small when doing the simulation. We used the empirical density function to simulate the real density function and we also need h to be small in order to expect a good approximation of the density function. But the graph seems to have too many modes when h is too small. So it is very crucial to have a appropriate h for simulating the density function. So before we validate the data, we have to find a value for h.

When we choose $h = 0.1$, we could see the distribution of our simulated standard Brownian motion is indeed normal. Additionally, we compute the mean and the variance of the data, and they all coincide with our model. *Data Validated!*

- The Classic Black Scholes Model
  $\pi_t^* = \frac{\mu_0 - r}{\sigma^2}$
- Technical Analysis
  Here we used a very simple technical analysis. It consists of resistance line and moving average. The strategy is that if we detect the resistance line, we will sell the risk asset when the price crosses the resistance line. Also we use moving average all the time.
- Strategies for the Newly Derived Model.
  - *In an upcrossing* $\pi_t^* = \frac{\mu_0 - r}{\sigma^2}$
  - *In a downcrossing* $\pi_t^* = \frac{\mu_0 - r}{\sigma^2} \frac{2\mu \frac{S(B_t^\mu)}{S(\varepsilon) - S(-\alpha)} \sum_{n=i_0+1}^{\infty} \alpha_n p^{n-1-i_0}}{\sigma(\sum_{n=0}^{i_0} \alpha_n + \frac{S(\varepsilon) - S(B_t^\mu)}{S(\varepsilon) - S(-\alpha)} \sum_{n=i_0+1}^{\infty} \alpha_n p^{n-1-i_0})}$

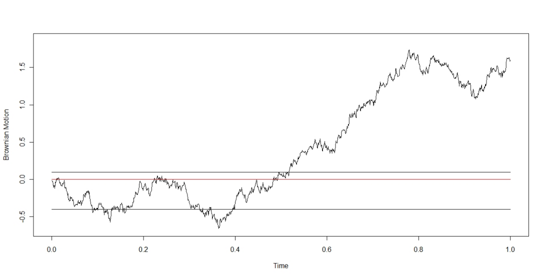
  where $S(x) = e^{-2\mu x}$, and $p = \frac{S(\epsilon) - S(0)}{S(\epsilon) - S(-\alpha)} \mathbb{P}_{-\alpha}(T_0 < \infty)$
    - ★ $\mathbb{P}_{-\alpha}(T_0 < \infty) = 1$, if $\mu \geq 0$
    - ★ $\mathbb{P}_{-\alpha}(T_0 < \infty) = e^{2\mu\alpha}$ otherwise.

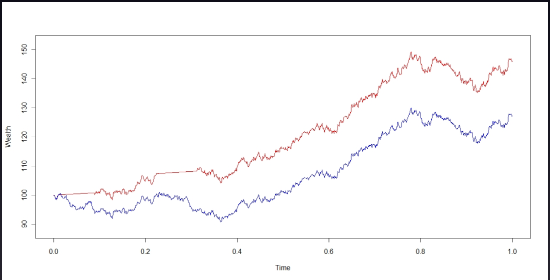
  Here, we assume the number of downcrossings is random.

# Assumptions

- All the parameters are known and fixed.
  $\mu_0 = 0.15, \ \sigma = 0.15, \ r = 0.02, \ \alpha = 0.5, \ \epsilon = 0.1$
- The resistance level is known
- At this stage, we do not allow short sell or borrow money from the riskless asset. i.e We always project the strategy onto the interval $[0, 1]$.

# A sample path



# The case we know exactly the distribution of the number of downcrossings

Case 1, the real law and the simulating law are the same, the number of downcrossings is fixed at 3.

| | Classic BS | Technical Analysis | Proposed strategy |
|---|---|---|---|
| mean | -0.000760 | 0.001788 | 0.005567 |
| SD | 0.061723 | 0.048606 | 0.046770 |

# The case with wrong law

Case 2, the real law of downcrossings has compact support. $\{1,2,3,4,5\}$with probabilties$\{0.1,0.2,0.4,0.2,0.1\}$ respectively. And the simulating law is a fixed number at 3.

| | Classic BS | Technical Analysis | Proposed strategy |
|---|---|---|---|
| mean | -0.000245 | 0.002315 | 0.007229 |
| SD | 0.063405 | 0.047997 | 0.049514 |

# Another case when we have a wrong law

Case 3, the real law of downcrossings has compact support. $\{1,2,3,4,5\}$with probabilties$\{0.1,0.1,0.2,0.2,0.4\}$ respectively. And the simulating law is a fixed number at 3.

| | Classic BS | Technical Analysis | Proposed strategy |
|---|---|---|---|
| mean | -0.000155 | 0.002181 | 0.006611 |
| SD | 0.062003 | 0.050446 | 0.049087 |

# Conclusion

With perfect knowledge of the parameters, we have the result that the final wealth of the proposed strategy is overwhelming over the other two strategies. Even with wrong laws of downcrossing numbers as long as it is close to the real one, the proposed strategy works still the best among the three.

- S. Achelis, *Technical analysis from A to Z.* McGraw Hill 2000.
- A. Borodin and P. Salminen, *Handbook of Brownian Motion - Facts and Formulae*. Probability and Its Applications, Birkhäuser Verlag, Basel, 1996.
- D. Lamberton and B. Lapeyre, *Stochastic Calculus Applied to Finance*. Chapman & Hall/CRC, Boca Raton, 1996.
- B.B. Bergery, C. Propheta and E. Tanré, Mathematical Model for Resistance and Optimal Strategy. No. 524 in *FINRISK working paper series*, March 2009.