Bifurcation Theory, a.y. 2013/14

Homework #5: Multiple Scale Method

January 26, 2014

Exercise 1:

Consider again the system in Excercise 1 of Homework #3, and obtain the bifurcation equation via the Multiple Scale Method:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} \mu - 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} x^3 + \alpha x z \\ x^2 \\ 0 \end{pmatrix}$$

Exercise 2:

Consider again the system in Excercise 2 of Homework #3, and obtain the bifurcation equation via the Multiple Scale Method:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \mu & 1 & 0 & 0 \\ -1 & \mu & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix} + \begin{pmatrix} x^3 \\ \alpha y v \\ x y \\ 0 \end{pmatrix}$$

Exercise 3:

Use the Multiple Scale Method to get the bifurcation equation of this system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \\ \dot{u} \end{pmatrix} = \begin{pmatrix} \mu & 1 & 0 & 0 \\ -1 & \mu & 0 & 0 \\ 0 & 0 & \nu & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ z \\ u \end{pmatrix} + \begin{pmatrix} \alpha x^3 + \beta y^3 \\ x z^2 \\ z y^2 + z u \\ x^2 \end{pmatrix}$$

where μ, ν are bifurcation parameters. Comment the type of bifurcation and the role active/passive of the state variables. [Hint: rescale μ, ν in such a way they appear in the perturbation equations at the same level than cubic nonlinearities].

Exercise 4:

Use the Multiple Scale Method to get the bifurcation equation of this system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \mu & 1 & 0 & 0 \\ -1 & \mu & 0 & 0 \\ 0 & 0 & \nu & \omega \\ 0 & 0 & -\omega & \nu \end{pmatrix} \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix} + \begin{pmatrix} x^3 + y^2 v \\ yu^2 \\ v^3 + y^3 \\ ux^2 \end{pmatrix}, \qquad \omega \neq 1, 2, 3, \ldots$$

where μ, ν are bifurcation parameters. Comment the type of bifurcation and the role active/passive of the state variables.

Exercise 5:

Repeat the Exercise 4 with $\omega = 3 + \sigma$, where σ is a small detuning parameter.