Bifurcation Theory, a.y. 2013/14

Homework #2: Perturbation Methods Part2

November 13, 2013

Exercise 1:

Consider the following linear non-homogeneous problem. Find the compatibility conditions under which the solution exist and it is bounded in time, and express this solution, (a) in the given basis, (b) in the basis of the eigenvectors of the Jacobian matrix:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix} e^{it} + \begin{pmatrix} c_1 \\ c_2 \\ c_3 \end{pmatrix}$$

Exercise 2:

With regard the 3-dimensional system below, apply the Multiple Scale Method to find the first- and the second-order amplitude modulation equations (namely $d_k A = F_k(A)\,,\ k = 1, 2$), and then recombine them according to the reconstitution method (to get $\dot{A} = F(A)$):

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \mu & 1 & 0 \\ -1 & \mu & 0 \\ 0 & 0 & -1 \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} xz \\ yz \\ x^2 + y^2 \end{pmatrix}$$

[Hint: perform the rescaling $(x, y, z) \to \varepsilon$, $\mu \to \varepsilon^2 \mu$; introduce $t_k = \varepsilon^k t,\ k = 0, 1, 2$ and then expand the state-variables as $x = x_0 + \varepsilon x_1 + \varepsilon^2 x_2 + \ldots$ and analogous].

Exercise 3:

Obtain a first-order amplitude modulation equation for the following system:

$$\frac{d}{dt}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{bmatrix} \mu & 1 & 0 \\ -1 & \mu & 0 \\ 0 & 0 & \mu \end{bmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} xz \\ yz \\ x^2 + y^2 \end{pmatrix}$$

[Hint: when $\mu = 0$, the Jacobian admits 'internally resonant' eigenvalues $\pm 1, 0$ which must *all* be accounted for in the generating solution].