Bifurcation Theory, a.y. 2013/14

Homework #3: Center Manifold Method

January 7, 2014

Exercise 1:

Consider the following nonlinear dynamical system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{z} \end{pmatrix} = \begin{bmatrix} \mu - 1 & 0 & 0 \\ 0 & -1 & 1 \\ 0 & -1 & -1 \end{bmatrix} \begin{pmatrix} x \\ y \\ z \end{pmatrix} + \begin{pmatrix} x^3 + \alpha x z \\ x^2 \\ 0 \end{pmatrix}$$

1. Find the critical value μ_c of μ at which a bifurcation occurs, and classify it [statical or dynamical?]
2. Apply the Center Manifold (CM) method to find the bifurcation equation describing the nonlinear dynamics when μ is close to μ_c [use a lower-order power expansion to describe the CM]
3. Plot the bifurcation diagram x vs μ and discuss its dependence on the parameter α.

Exercise 2:

Consider the following nonlinear dynamical system:

$$\begin{pmatrix} \dot{x} \\ \dot{y} \\ \dot{u} \\ \dot{v} \end{pmatrix} = \begin{pmatrix} \mu & 1 & 0 & 0 \\ -1 & \mu & 0 & 0 \\ 0 & 0 & -1 & 1 \\ 0 & 0 & -1 & -1 \end{pmatrix} \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix} + \begin{pmatrix} x^3 \\ \alpha y v \\ x y \\ 0 \end{pmatrix}$$

1. Find the critical value μ_c of μ at which a bifurcation occurs, and classify it [statical or dynamical?]
2. Apply the Center Manifold (CM) method to find the bifurcation equation describing the nonlinear dynamics when μ is close to μ_c [use a lower-order power expansion to describe the CM]
3. Plot the bifurcation diagram $a := \sqrt{x^2 + y^2}$vs μ and discuss its dependence on the parameter α.