Bifurcation Theory, a.y. 2013/14

Homework #3: Normal Form Theory

January 20, 2014

Exercise 1:

Find the Normal Form of the following equation, describing the dynamics at a Hopf bifurcation:

$$\dot{x} = i\omega x + \alpha_1 x\bar{x} + \alpha_2 x^2 + \alpha_3 x^2 \bar{x}$$

Here the state variable x and the coefficients α are complex, while ω is real; the bar denotes the complex conjugate.

Exercise 2:

Consider the system of the Exercise 1, in which a bifurcation parameter $\mu \in \mathbb{R}$ has been added:

$$\dot{x} = (\mu + i\omega)\, x + \alpha_1 x\bar{x} + \alpha_2 x^2 + \alpha_3 x^2 \bar{x}$$

Find the Normal Form of this equation [hint: uses the 'suspension trick', by appending the trivial equation $\dot{\mu} = 0$].

Exercise 3:

Find the Normal Form of the following equations, describing the dynamics at a Takens-Bogdanov (double zero) bifurcation:

$$\begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} + \begin{pmatrix} \alpha_1 x_2^3 + \alpha_2 x_1 x_2^2 \\ \alpha_3 x_1^3 + \alpha_4 x_1^2 x_2 \end{pmatrix}$$

Here all quanities are real.