APPLIED PARTIAL DIFFERENTIAL EQUATIONS (MATHMODS)

RAMÓN G. PLAZA

Lectures: Mondays, Wednesdays 14:30 - 17:30 hrs. Room: A-1.7

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Office hours: Tuesday 16:30 - 18:30 hrs. (office 1041 (Marini)).

Exams:

- First midterm exam: Wednesday, November 7. 2:30pm - 5:30pm
- Second midterm exam: Friday, December 14. 2:30pm - 5:30pm

Syllabus

1. **First order equations.** Examples: population growth (McKendrick equation) and continuous traffic flow (Lighthill-Whitham-Richards equation). Method of characteristics. Local solutions to linear and quasilinear equations. The transport equation. Burgers' equation. Shocks and rarefaction waves. Riemann problem for scalar conservation laws.
2. **Second order equations.** Classification. Canonical form. Well posed problems.
3. **The heat equation.** Derivation and diffusive phenomena. The fundamental solution (Poisson kernel). The Fourier transform method. Initial-boundary-value problems. Maximum principle and uniqueness. Non homogeneous problem: Duhamel principle. Regularity. Applications to diffusion, random walks, problems in finance.
4. **The Laplace equation.** Derivation (elastic membranes, electrostatic problems, fluid dynamics). Laplace and Poisson equations. Fundamental solution and Green's function. Poisson formula. Maximum principle. Dirichlet problem. Properties of harmonic functions. Existence of solution to the Dirichlet problem: Perron's method. Energy methods and uniqueness. Dirichlet principle (variational formulation).
5. **The wave equation.** Derivation (small vibrations, elastic membrane). Wave equation in $\mathbb{R}$, Cauchy problem and D'Alembert's formula. Fixed boudary points. Non-homogeneous problem. Wave equation in $\mathbb{R}^d$. Light cone and the method of spherical means. Fundamental solution ($d = 3$) and Huygens'principle. The method

of descent of Hadamard ($d = 2$). Non-homogeneous problems and Duhamel's principle. Energy methods. Introduction to nonlinear waves.

6. **Symmetric hyperbolic systems*.** Examples (transmission lines, shallow water waves, Maxwell equations). Energy estimates and uniqueness of solutions. Initial boundary value problems. Introduccion to Kreiss-Métivier conditions for well-posedness.

Bibliography

Main references. The three main texts that I will use during this course will be: the book by Salsa [9], the first part of the book by Evans [2], and the classic text of John [6]. I will mainly follow Salsa's book, although I will also strongly recommend some selected readings from the other two texts.

Supplementary bibliography. The book by Strauss [10] is a good introductory text to PDEs which could be interesting from the applications viewpoint. Another excellent reference is the text by Renardy and Rogers [8]. The first two chapters of the notes by Han and Lin [5] constitute an excellent (and not complicated) complement to section 4.

Advanced bibliography. From the advanced material that the interested student may consult I suggest: the second volume of Courant and Hilbert [1], the book by Taylor [11] and of course, Folland's text [3]. An excellent reference for elliptic equations is the advanced book by Gilbarg and Trudinger [4]. An introduction to modern methods in PDEs as well as the nonlinear theory can be found in the second part of Evans book [2]. A modern text, recommended as a second reading (after a basic course of PDE), is the book by Jost [7].

References

[1] R. Courant and D. Hilbert, *Methods of mathematical physics. Vol. II: Partial differential equations*, Wiley Classics Library, John Wiley & Sons Inc., New York, 1989. Reprint of the 1962 original, A Wiley-Interscience Publication.

[2] L. C. Evans, *Partial Differential Equations*, vol. 19 of Graduate Studies in Mathematics, Amer. Math. Soc., Providence, RI, 1998.

[3] G. B. Folland, *Introduction to Partial Differential Equations*, Princeton University Press, Second ed., 1995.

[4] D. Gilbarg and N. S. Trudinger, *Elliptic partial differential equations of second order*, Classics in Mathematics, Springer-Verlag, Berlin, 2001. Reprint of the 1998 edition.

[5] Q. Han and F. Lin, *Elliptic partial differential equations*, vol. 1 of Courant Lecture Notes in Mathematics, New York University Courant Institute of Mathematical Sciences, New York, 1997.

[6] F. John, *Partial Differential Equations*, vol. 1 of Applied Mathematical Sciences, Springer-Verlag, New York, Fourth ed., 1982.

*If time permits.

[7] J. Jost, *Partial differential equations*, vol. 214 of Graduate Texts in Mathematics, Springer, New York, second ed., 2007.
[8] M. Renardy and R. C. Rogers, *An introduction to partial differential equations*, vol. 13 of Texts in Applied Mathematics, Springer-Verlag, New York, second ed., 2004.
[9] S. Salsa, *Partial differential equations in action. From modelling to theory*, Universitext, Springer-Verlag Italia, Milan, 2008.
[10] W. A. Strauss, *Partial differential equations. An introduction*, John Wiley & Sons Inc., New York, 1992.
[11] M. E. Taylor, *Partial differential equations. Basic theory*, vol. 23 of Texts in Applied Mathematics, Springer-Verlag, New York, 1996.

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