LAPLACE TRANSFORM $\quad L(f)(p) = F(p) = \int_0^{+\infty} e^{-pt} f(t)\,dt$

$$L(1) = \frac{1}{p} \qquad L(t^n) = \frac{n!}{p^{n+1}} \qquad L(e^{at}) = \frac{1}{p-a} \qquad L(\operatorname{sen} kt) = \frac{k}{k^2+p^2} \qquad L(\cos kt) = \frac{p}{p^2+k^2}$$

$$L(f^{(n)}(t)) = p^n F(p) - p^{n-1} f(0) - p^{n-2} f'(0) - \dots - f^{(n-1)}(0)$$

$$F^{(n)}(p) = (-1)^n L(t^n f(t))$$

$$L\left(\int_0^t f(s)\,ds\right) = \frac{F(p)}{p} \qquad L\left(\frac{f(t)}{t}\right) = \int_p^{+\infty} F(u)\,du$$

$$L(f(t-\tau)H(t-\tau)) = e^{-\tau p} F(p)$$

$$L(e^{p_0 t} f(t)) = F(p-p_0)$$

FOURIER TRANSFORM $\quad F(f(x))(\omega) = \hat{f}(\omega) = \int_{-\infty}^{+\infty} e^{-2\pi i \omega x} f(x)\,dx$

$$F(e^{-ax^2}) = e^{\frac{-\pi^2\omega^2}{a}} \sqrt{\frac{\pi}{a}} \qquad F\left(\frac{1}{1+x^2}\right) = \pi e^{-\pi 2|\omega|}$$

$$\underset{a>0}{F(e^{-a|x|})} = \frac{2a}{a^2+4\pi^2\omega^2} \qquad F(f(x+h))(\omega) = e^{2\pi\omega h i} F(f(x))(\omega)$$

$$F(e^{-2\pi i h x} f(x)) = F(f(x))(\omega+h)$$

$$F\left(f\left(\frac{x}{A}\right)\right)(\omega) = |A| F(f(x))(A\omega) \qquad F(\overline{f(x)})(\omega) = \overline{\hat{f}(-\omega)}$$

$$\hat{f}^{(n)}(\omega) = (-2\pi i)^n (x^n f(x))^\wedge(\omega)$$

$$F(f^{(n)}(x))(\omega) = (2\pi i\omega)^n \hat{f}(\omega)$$

Complex Analysis Test (3CF)

a.a. 2014/15

Ex 1 [10] Compute, by definition, the Fourier Transform of $f(x)=\dfrac{x^2}{(x^2+1)(x^2+4)}$, if it is possible.

Ex 2 [10] Solve applying Laplace Transform

$$\begin{cases} x'(t) = y(t) - 1 & x(0) = 3 \\ y'(t) = 2x(t)' + y(t) & y(0) = 5 \end{cases}$$

Ex 3 [10] Find the Laurent expansion, centred in $z_0=0$, converging in $z_1=\frac{1}{3}+i\frac{2}{\sqrt{3}}$, for

$$f(z) = \frac{1}{(z^2+1)(z-2)}$$

Complex Analysis Test a.a 2013/14

Ex1

Solve by Fourier Transform

$$\begin{cases} u_t - 2u_{xx} = 0 & t > 0, \ t \in \mathbb{R} \\ u(x,0) = e^{-3x^2} \end{cases}$$

Ex 2 Compute applying the Residues theorem

$$\int_{-\infty}^{\infty} \frac{\operatorname{sen} x}{(x-1)(x^3+1)} \, dx$$

Ex 3 Solve by Laplace Transform

$$\begin{cases} y''(t) = y'(t) + 2y(t) + 12e^{-2t} \\ y(0) = 4, \ y'(0) = -6 \end{cases}$$

Ex 4 Find the Laurent expansion, centred in $z_0 = 2$, converging in $z_1 = 0$, for

$$f(z) = \frac{z-3}{(z-1)(z+1)(z-2)}$$

Final Test on Complex Analysis

a a 2012/13

Ex 1 Solve by means of Laplace Transform

$$\begin{cases} x'(t) = y(t) \\ y'(t) = -x(t) + 2\sin(t) \\ x(0) = 0 \quad y(0) = 0 \end{cases}$$

Ex 2 Compute, by definition, the Fourier Transform of

$$f(x) = \frac{1}{x^2+6x+10}$$

Ex 3 Determine and classify the singularities of $f(z) = \dfrac{(z-1)^2}{\sin^3(z-1)}$

and compute Residue$(f, z=1)$

Ex-Extra Solve for $t \geq 0$

$$\begin{cases} y'(t) + \int_0^t (t-z)\,y(z)\,dz = b(t) \\ y(0) = 0 \end{cases}$$

where $b(t) = \begin{cases} -1 & 0 \leq t \leq 1 \\ 1 & 1 \leq t \leq 2 \\ e^{-(t-2)} & t \geq 2 \end{cases}$

Mid-term Test of Complex Analysis

a.a. 2012/13

Ex 1 Solve the equation and draw a picture of the set of solutions

$$\mathrm{Re}\left(\frac{z-2i}{2z+i}\right)=0 \qquad z\in\mathbb{C}$$

Ex 2 Find the holomorphic function whose real part is

$$u(x,y)=x^2-y^2+e^{-x}\cos y$$

Ex 3 Discuss, in term of the radius r of the circle $|z|=r$, the value of

$$\int_\gamma \frac{5z^2-3z+2}{(z-1)^3}\,dz$$

Ex 4 Find the Laurent expansion, centred in $z_0=-1$, converging in $z_1=3-i$, for

$$f(z)=\frac{z}{(z-1)(z+1)^2}$$

Ex-Extra Prove that, if $f:\mathbb{C}\to\mathbb{R}$ has complex derivative in z_0, then $f'(z_0)=0$