APPLIED PARTIAL DIFFERENTIAL EQUATIONS

MIDTERM EXAM
31 OCTOBER 2011

Duration (total): 90 minutes.

Family and first name: ______________________________

Problem 1. Wave equation

$$u_{tt} - 4u_{xx} = 0, \tag{0.1}$$

(1) Derive D'Alembert formula with initial data

$$u|_{t=0} = \cos x, \quad u_t|_{t=0} = x^2,$$

and for $x \in (0, +\infty)$ solve the half line problem of (0.1) with given data

$$u_x|_{x=0} = 0, \qquad t > 0.$$

(2) $x \in (0,1)$, $t > 0$, using separation of variable solve the initial boundary value problem of (0.1) with given data

$$u|_{t=0} = x^2 - 2x, \qquad u_t|_{t=0} = 0,$$
$$u|_{x=0} = u_x|_{x=1} = 0, \qquad t > 0.$$

Problem 2. Find the unique weak entropy solution of the following Cauchy problem

$$u_t + (q(u))_x = 0, \quad x \in \mathbb{R}$$
$$u|_{t=0} = g(x)$$

with

$$q(u) = 2 - u^2, \quad g(x) = \begin{cases} 0 & x \leq 0 \\ 1 & 0 < x < 1 \\ 2 & x \geq 1 \end{cases}$$

Draw the characteristic lines in (x,t) space and draw the graph of solution at $t = 1$ in (x,u) space.